

FURTHER MATHS

SUMMER WORK

2024

Section	Topic	Score	Review		
1.1	Rearranging formulae	/32	😊	😐	😞
1.2	Turning points	/25	😊	😐	😞
1.3	Exponential graphs	/14	😊	😐	😞
1.4	Simultaneous equations	/38	😊	😐	😞

Approximate completion time: 3 hours

Deadline: Friday 2nd August 2024

Please email a photo of this completed front page to
jonesn@wallingfordschool.com.

Be honest with your marking and complete the review of how confident you are with the work.

Please bring your completed questions to your first lesson in September, I am expecting to see full workings in your written work.

1.1 Rearranging formulae:

Video search: "Maths genie changing the subject 2"

Link: <https://www.mathsgenie.co.uk/changing-the-subject2.html>

Question 1

Rearrange to make x the subject.

(4 marks)

$$y = \frac{8(w-x)}{x}$$

Question 2

Make m the subject of

(4 marks)

$$f = \frac{4-3m}{5+m}$$

Question 3

Make r the subject of

(4 marks)

$$m = \sqrt{\frac{6a+r}{5r}}$$

Question 4

Make x the subject of:

(4 marks)

$$y = \frac{a^2 x + b}{2-x}$$

Question 5

Make m the subject of

(4 marks)

$$\frac{m}{v} - \frac{t}{b} = \frac{m-t}{R}$$

Question 6

Make x the subject of the following, **giving the larger of the two expressions you obtain:** (4 marks)

$$y = \frac{x^2+1}{x+1}$$

(Hint at end of paper)

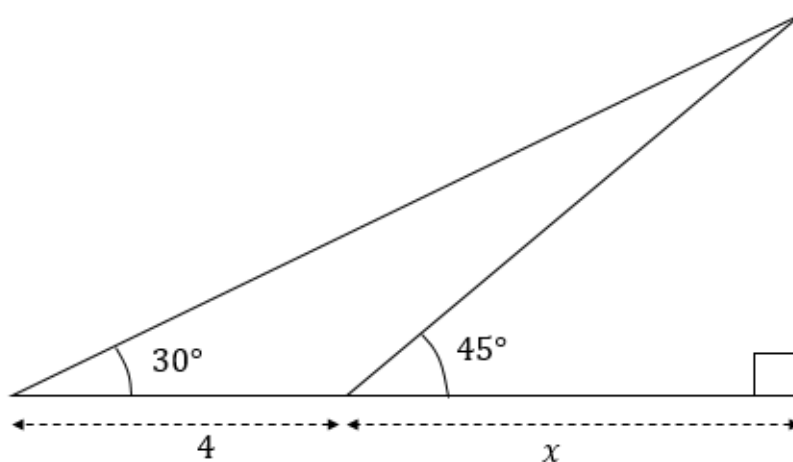
Question 7

Make x the subject of the following, **fully simplifying your answer.** (4 marks)

$$y + 2 = \frac{x+1}{x-y}$$

Question 8

Determine the **exact** value of x . (4 marks)



1.2 Turning points:

Video Search: “Maths genie completing the square”

Link: <https://www.mathsgenie.co.uk/completing-the-square.html>

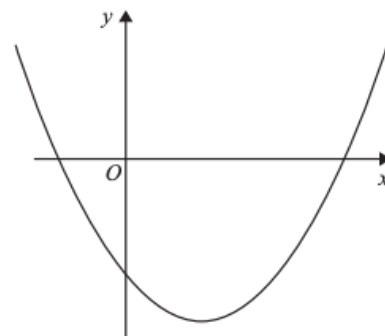
Question 1

Here is a sketch of a curve.

The equation of the curve is $y = x^2 + ax + b$ where a and b are integers.

The points $(0, -5)$ and $(5, 0)$ lie on the curve.

Find the coordinates of the turning point of the curve.



(2 marks)

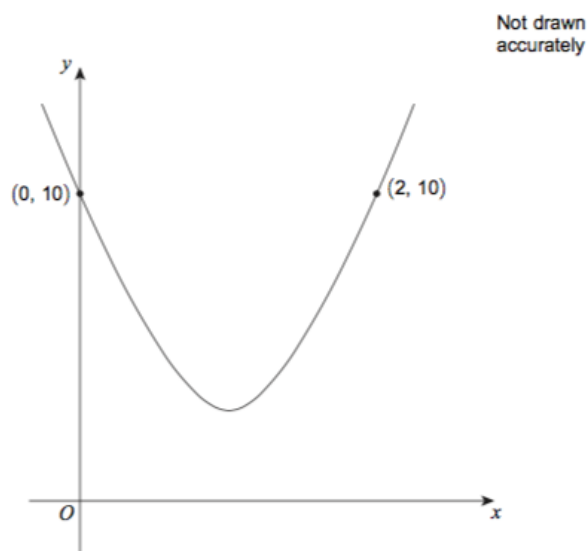
(Hint at end of paper)

Question 2

The sketch shows the quadratic curve $y = 4(x - a)^2 + b$

The curve passes through $(0, 10)$ and $(2, 10)$

Work out the values of a and b .



(4 marks)

Question 3

The point $(-3, -4)$ is the turning point of the graph of $y = x^2 + ax + b$, where a and b are integers.

Find the values of a and b .

(3 marks)

(Hint at end of paper)

Question 4

Find the coordinates of the minimum point of the graph with equation

$$y = 2x^2 + 12x - 2$$

(4 marks)

Question 5

Find the coordinates of the minimum point of the graph with equation

$$y = 3x^2 - 6x - 1$$

(4 marks)

Question 6

Find the coordinates of the maximum point of the graph with equation

$$y = -4x^2 - 4x - 2$$

(4 marks)

Question 7

The minimum point of a graph is $(2, 0)$.

Given that the graph crosses the y -axis at $(0, 6)$, find an equation of the graph.

(4 marks)

(Hint at end of paper)

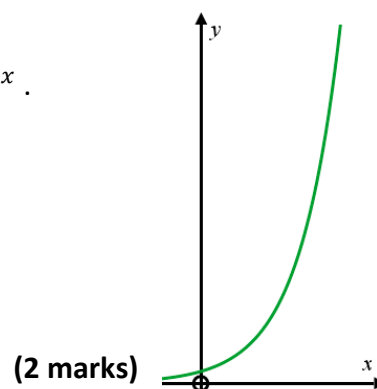
1.3 Exponential Graphs:

Video search: "Finding an exponential function through 2 points"

Question 1

The graph shown is exponential and its equation is in the form $y = a^x$.

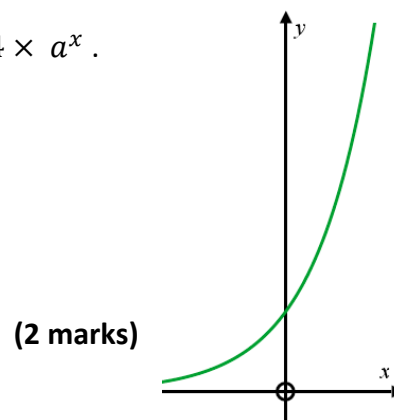
Given that $(2,25)$ lies on the graph, find the value of a .



Question 2

The graph shown is exponential and its equation is in the form $y = 4 \times a^x$.

Given that $(2,144)$ lies on the graph, find the value of a .



Question 3

An exponential graph has equation $y = a \times 2^x$.

The point with coordinates $(1,10)$ lies on the curve.

Find the value of a .

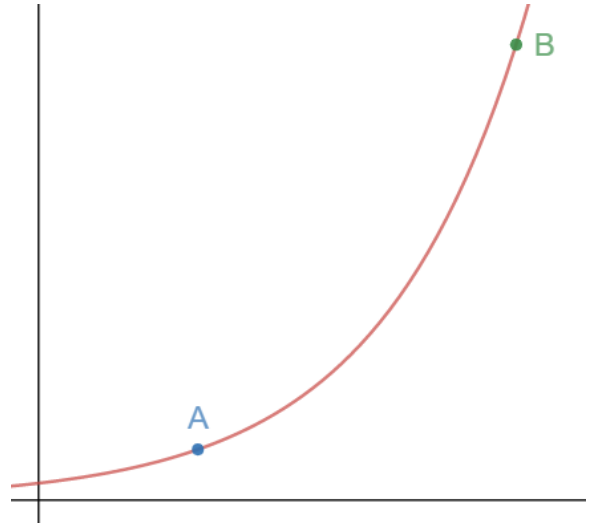
(2 marks)

Question 4

The graph shown is of an equation of the form;

$$y = a \times b^x$$

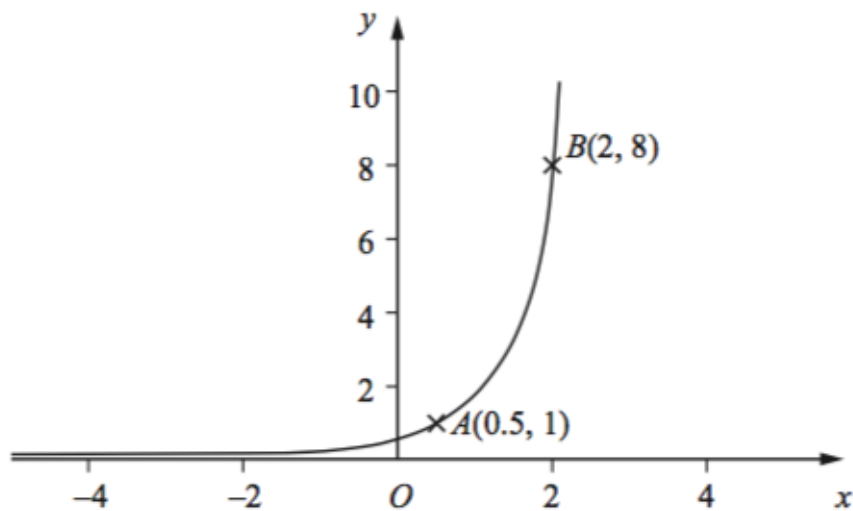
A is the point (1,6) and B is the point (3,54).



Find the equation of the line by determining the values of a and b.

(4 marks)

Question 5



The diagram shows a sketch of the graph $y = ab^x$. The curve passes through the points A (0.5, 1) and B (2, 8).

The point C (-0.5, k) lies on the curve.

Find the value of k .

(4 marks)

1.4 Simultaneous Equations

Video search: “Maths genie simultaneous equations with a quadratic”

Link: <https://www.mathsgenie.co.uk/simultaneous-quadratic.html>

Question 1

Solve algebraically these simultaneous equations.

$$y = x^2 + 5x - 4$$

$$y = 8 - 3x$$

Give your answers correct to 2 decimal places.

(5 marks)

Question 2

Solve the simultaneous equations

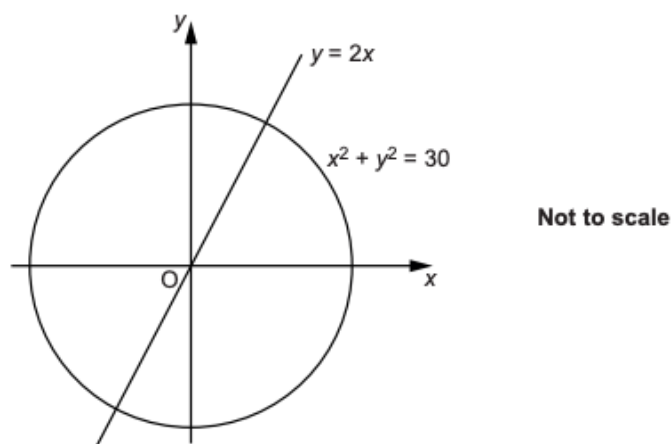
$$y = 2x - 3$$

$$x^2 + y^2 = 2$$

(6 marks)

Question 3

Find the exact coordinates of the two intersections of the line $y = 2x$ and the circle $x^2 + y^2 = 30$.



(5 marks)

Question 4 Challenge

Solve the pair of simultaneous equations

$$x^2 + 3y = 10 \text{ and}$$

$$3 + y = \frac{10}{x}$$

(6 marks)

Hint: To find solutions to a cubic equation = 0, substitute in factors of the constant term (the one without an x). If they make the equation = 0 they are a value of x. Don't forget to try + and - values.

Question 5 Challenge

Two numbers x and y are such that $x + y = 20$ and $\frac{1}{x} + \frac{1}{y} = \frac{1}{2}$.

What is the value of $x^2y + xy^2$?

(6 marks)

Hint: You don't need to find the values of x and y here. Factorise the final equation to find the expressions you need the values for.

Hints

1.1 Question 6 hint:

The problem with this question is that you have a term in x^2 . When you have multiplied up you now need to form a quadratic = 0. Solve this using the quadratic formula treating ys like numbers.

1.2 Question 1 hint:

Substitute the points into the quadratic $y = x^2 + ax + b$ to form a pair of simultaneous equations

1.2 Question 3 hint:

Substitute your turning point into the completed square form for a quadratic $y = (x + p)^2 + q$ and then multiply out to form your equation.

1.2 Question 7 hint:

This is almost the same as question 3 but you will need the form $y = a(x + p)^2 + q$ instead.

1.1 Answers

Question 1

$$x = \frac{8w}{y+8}$$

$yx = 8(w-x)$ or $y = \frac{8w-8x}{x}$	M1	
$yx = 8w - 8x$	M1dep	oe eg $yx - 8w + 8x = 0$ Implies M1 M1
$yx + 8x = 8w$ or $x(y+8) = 8w$ or $\frac{8w}{y+8}$	M1dep	oe dep on M1 M1 Implies M1 M1 M1
$x = \frac{8w}{y+8}$	A1	oe eg $\frac{-8w}{-y-8}$ Must have $x =$ SC2 $x = \frac{8w}{y+1}$ SC1 $\frac{8w}{y+1}$

Question 2

$$m = \frac{4-5f}{f+3}$$

$m = \frac{4-5f}{f+3}$	4	M1 for multiplying both sides by $5+m$ as a first step M1 for correctly moving their m terms to one side and their other terms to the other side M1 for factorising A1 for $m = \frac{4-5f}{f+3}$ oe
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Question 3

$$r = \frac{6a}{5m^2-1}$$

$m^2 = \frac{6a+r}{5r}$ $m^2 \times 5r = 6a+r$ $5rm^2 - r = 6a$		$r = \frac{6a}{5m^2-1}$	4	M1 M1 M1 A1 or for $r = \frac{-6a}{1-5m^2}$ oe NB: to award A1 we must see $r = \frac{6a}{5m^2-1}$ in working if $\frac{6a}{5m^2-1}$ alone is given as answer
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Question 4

$$x = \frac{2y-b}{a^2+y}$$

M1 multiplying by $2-x$

M1 collecting x terms to one side

M1 isolating x terms

A1 correct answer

Question 5

$$m = \frac{tv(R-b)}{b(R-v)}$$

$$\frac{mb-tv}{vb} = \frac{m-t}{R}$$

$$mbR - tvR = m vb - tvb$$

$$mbR - mvb = tvR - tvb$$

$$m(bR - vb) = tvR - tvb$$

OR

$$\frac{m}{v} - \frac{t}{b} = \frac{m}{R} - \frac{t}{R}$$

$$\frac{m}{v} - \frac{m}{R} = \frac{t}{b} - \frac{t}{R}$$

$$\frac{mR - mv}{vR} = \frac{tR - tb}{bR}$$

$$\frac{m(R-v)}{vR} = \frac{tR - tb}{bR}$$

$$m = \frac{tv(R-b)}{b(R-v)}$$

4

M1 for putting LHS over a common denominator with at least one correct numerator (ignore signs) or for showing an intention to multiply each term on both sides by R or v or b
 M1 for rearranging correctly to isolate terms in m
 M1 for factorising with common factor m from 2 terms
 A1 for $m = \frac{tv(R-b)}{b(R-v)}$ oe

Question 6

$$x = \frac{y + \sqrt{y^2 - 4(1-y)}}{2}$$

M1 multiplying by $x + 1$

M1 forming a quadratic = 0 $x^2 - yx + (1 - y) = 0$

M1 attempting quadratic formula (or completing the square)

A1 Correct answer

Question 7

$$x = y + 1$$

Question 8

$$2 + 2\sqrt{3}$$

1.2 Answers

Question 1

(2, -9)

(2, -9)	P1	substitutes $x = 0, y = -5$ into $y = x^2 + ax + b$ ($b = -5$) or substitutes $x = 5, y = 0$ into $y = x^2 + ax + b$ ($0 = 25 + 5a + b$) or starts process to find other intercept, eg writes $y = (x - 5)(x - k)$
	P1	for complete process to find two intercepts, eg. substitutes the second point into $y = x^2 + ax + b$ and solves to find a ($= -4$) and b ($= -5$) or substitutes $x = 0, y = -5$ into $y = (x - 5)(x - k)$ and solves to find k ($= -1$)
	P1	(dep on P2) for factorising or completing the square of $x^2 + "-4" x + "-5"$ and identifying the x -coordinate of the turning point or for a complete process to find the x -coordinate of the turning point, eg $(5 + "-1")/2$
	A1	cao

Question 2

$$a = 1$$

(1 (or a) is) Midway between 0 and 2 or $\frac{2+0}{2} = 1$ or $\frac{2-0}{2} = 1$	B1	oe
Minimum point (at $x = 1$ (or $x = a$)) or Symmetrical (about $x = 1$ (or $x = a$))	B1	oe

$$b = 6$$

$10 = 4(0 - 1)^2 + b$ or $10 = 4(2 - 1)^2 + b$	M1	oe eg $10 = 4 + b$
6	A1	

Question 3

$$a = 6, b = 5$$

$$[a =] 6 \quad [b =] 5$$

3	M1 for $y = (x + 3)^2 - 4$
1 AO1.3b	M1 for multiplying out and simplifying
1 AO2.1a	<i>their</i> $y = (x + 3)^2 - 4$
1 AO3.1b	

Question 4

$$(-3, -20)$$

$$M1 \ 2[x^2 + 6x] - 2$$

$$M \ 1 \ A1 \ 2(x + 3)^2 - 20$$

Question 5

$$(1, -4)$$

$$M1 \ 3[x^2 - 2x] - 1$$

$$M \ 1 \ A1 \ 3(x - 1)^2 - 4$$

A1 Correct answer

Question 6

$$\left(-\frac{1}{2}, -1\right)$$

$$M1 \ -4[x^2 + x] - 2$$

$$M \ 1 \ A1 \ -4(x + 0.5)^2 - 1$$

A1 Correct answer

Question 7

$$y = \frac{3}{2}(x - 2)^2$$

$$M1 \ y = a(x - 2)^2 + 0$$

$$A1 \ a = 3/2$$

$$M1 \ 6 = a(-2)^2$$

A1ft correct answer

1.3 Answers

Question 1

$a = 5$ M1 Substitute values, A1 correct answer

Question 2

$a = 6$ M1 Substitute values, A1 correct answer

Question 3

$a = 5$ M1 Substitute values, A1 correct answer

Question 4

$a = 2, b = 3$

M1 $6 = ab$ and $54 = ab^3$

M1 division $b^2 = 9$

A1 $b = 3$, A1 $a = 2$

Question 5

$$k = \frac{1}{4}$$

$$\begin{aligned} 1 &= a \times b^{0.5}, 8 = a \times b^2 \\ \frac{a \times b^2}{a \times b^{0.5}} &= \frac{8}{1} \\ b^{\frac{3}{2}} &= 8, b = 4 \quad a = \frac{1}{2} \\ c &= \frac{1}{2} \times 4^{-\frac{1}{2}} \end{aligned}$$

$$k = \frac{1}{4}$$

4

M1 for $1 = a \times b^{0.5}, 8 = a \times b^2$

M1 for correct method to eliminate either a or b could be implied by

$$\frac{a \times b^2}{a \times b^{0.5}} = \frac{8}{1} \text{ or } \frac{b^2}{b^{0.5}} = 8 \text{ or } 8 = a \left(\frac{1}{a^2}\right)^2 \text{ oe}$$

$$\text{A1 for } b = 4 \quad a = \frac{1}{2}$$

$$\text{A1 for } k = \frac{1}{4} \text{ oe}$$

1.4 Answers

Question 1

$$x = 1.29, y = 4.13 \text{ or } x = -9.29, y = 35.87$$

$$x = 1.29, y = 4.13 \\ x = -9.29, y = 35.87$$

<p>5</p> <p>M2 for $x^2 + 8x - 12 [= 0]$ Or M1 for attempt to equate Eg $8 - 3x = x^2 + 5x - 4$</p> <p>AND M1FT for substitution into quadratic formula $\frac{-8 \pm \sqrt{8^2 - 4 \times -12}}{2}$</p> <p>A1 for $x = 1.29, x = -9.29$ AND A1 for $y = 4.13, y = 35.87$</p> <p>After A0 allow SC1 for one pair of x and y values correct or for both y values correctly FT their x values substituted into $y = 8 - 3x$</p>	<p>FT <i>their</i> quadratic equation, condone one error dependent on at least M1</p> <p>Allow A marks if solutions are clear in working, but transferred to wrong places on answer lines</p>
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Question 2

$$x = 1, y = -1 \text{ or } x = \frac{7}{5}, y = -\frac{1}{5}$$

$x^2 + (2x - 3)^2 = 2$		6	M1 for correct substitution
$x^2 + 4x^2 - 6x - 6x + 9 = 2$ or $x^2 + 4x^2 - 12x + 9 = 2$			B1 (indep) for correct expansion of $(2x - 3)^2$ even if unsimplified
$5x^2 - 12x + 7 (= 0)$			B1 for correct simplification Condone omission of '= 0'
$(5x - 7)(x - 1) (= 0)$ or $\frac{12 \pm \sqrt{4}}{10}$ or $\frac{12 \pm \sqrt{4}}{10}$ or $\frac{6 \pm 1}{5}$			B1 for correct factorisation or for correct substitution into quadratic formula and correct evaluation of ' $b^2 - 4ac$ ' or for using square completion correctly as far as indicated
$x = 1$ or $x = \frac{7}{5}$			A1 for both values of x dep on all preceding marks
	$x = 1, y = -1$ $x = \frac{7}{5}, y = -\frac{1}{5}$		A1 for complete, correct solutions (need not be paired) dep on all preceding marks No marks for $x = 1, y = -1$ with no working

Question 3

$$x = \sqrt{6}, y = 2\sqrt{6} \text{ and } x = -\sqrt{6}, y = -2\sqrt{6}$$

$$\text{M1 } x^2 + 4x^2 = 30$$

$$\text{M1 } x^2 = 6$$

$$\text{A1 } x = \sqrt{6} \text{ and A1 } x = -\sqrt{6}$$

A1 Correct answer

Question 4

$$x = 3, y = \frac{1}{3} \text{ or } x = 2, y = 2 \text{ or } x = -5, y = -5$$

$$\text{M1 } x^2 + 3\left(\frac{10}{x} - 3\right) = 10$$

$$\text{A1 } x^3 - 19x + 30 = 0$$

M1 attempts to find x by substitution of a factor.

A1 any one of $x = 3, 2,$ or -5

A1 all of $x = 3, 2,$ or -5

A1 correct answer

Question 5

800

We have that $\frac{1}{2} = \frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy} = \frac{20}{xy}$. So $\frac{1}{2} = \frac{20}{xy}$ and hence $xy = 40$. Therefore

$$x^2y + xy^2 = xy(x+y) = 40 \times 20 = 800.$$

M1 attempts to combine fractions

A1 correct combined fractions $\frac{x+y}{xy} = \frac{1}{2}$

A1 correct factorization of target function $xy(x+y)$

M1 attempts to find value of xy

A1 correct value of $xy = 40$

A1 correct answer of 800