FURTHER MATHS SUMMER WORK 2024

Section	Topic	Score	Review		
2.1	Complex numbers part 1	/37	☺	😐	☹
2.2	Complex numbers part 2	/16	☺	😐	☹

Approximate completion time: 2 hours

Deadline: Wednesday 4th September 2024

Be honest with your marking and complete the review of how confident you are with the work.

Please bring your completed questions to your first lesson in September, I am expecting to see full workings in your written work.

2.1 Complex Numbers

Video search: **"TLMaths – The complex conjugate"**
Videos: **B3 – 01, B3 – 02**
Video search: **"TLMaths – Working with complex numbers"**
Videos: **B2 – 04, B2 – 05**

Question 1

Given that $z = 2 + 6i$, write down the value of z^* , where z^* is the complex conjugate of z .

(1 mark)

Question 2

Given that $z = 1 - 10i$, write down the value of z^* , where z^* is the complex conjugate of z .

(1 mark)

Question 3

Write $(i + 5)(2i - 3)$ in the form $a + bi$.

(2 marks)

Question 4

A complex number x is given by $z = a + 2i$ where a is a non-zero real number.

a) Find, in terms of a, $z^2 + 2z$

b) Given that $z^2 + 2z$ is real, find the value of a .

(3 marks)

Question 5

The complex numbers z_1 and z_2 are given by $z_1 = 2 - 3i$ and $z_2 = a + 4i$ where a is a real number.

Given that $z_1 z_2 = (z_1 z_2)^*$ find the value of a .

(2 marks)

Question 6

Write $(4-3i)^2$ in the form $a+bi$.

(2 marks)

Question 7

Express $(2+3i)^3$ in the form $a+ib$.

(3 marks)

Question 8

$z=2-3i$ Find z^2 in its simplest form.

(2 marks)

Question 9

$z=2-i\sqrt{3}$ Use algebra to express $z+z^2$ in the form $a+bi\sqrt{3}$, where a and b are integers.

(3 marks)

Question 10

$z=\frac{4}{1+i}$ Find z in the form $a+ib$ where

(2 marks)

Question 11

The complex numbers z_1 and z_2 are given by

$$z_1=2-i \quad \text{and} \quad z_2=-8+9i$$

Find $\frac{z_2}{z_1}$ in the form $a+bi$, where a and b are real.

(3 marks)

Question 12

Given that $z = 2 - i$, use algebra to express $\frac{z}{z^*}$ in the form $a + bi$, where a and b are constants to be found, and where z^* is the complex conjugate of z .

(3 marks)

Question 13

Given that $z = 4 - i$, use algebra to express $\frac{z+2}{z-3}$ in the form $a + bi$, where a and b are constants to be found.

(3 marks)

Question 14

$z = 2 - i\sqrt{3}$

Use algebra to express $\frac{z+7}{z-1}$ in the form $c + di\sqrt{3}$, where c and d are integers.

(4 marks)

Question 15

The complex number w is given by $w = \frac{p-4i}{2-3i}$ where p is a real constant.

Express w in the form $a + bi$, where a and b are real constants. Give your answer in its simplest form in terms of p .

(3 marks)

2.2 Complex Numbers

Question 1

Given that $\frac{3w+7}{5} = \frac{p-4i}{3-i}$ where p is a real constant. Express w in the form $a + bi$, where a and b are real constants. Give your answer in its simplest form in terms of p.

(5 marks)

Question 2

$$z_1 = 2 + 3i\ ,\quad z_2 = 3 + 2i\ ,\quad z_3 = a + bi\ ,$$

Given that $w = \frac{z_1 z_3}{z_2}$, find w in terms of a and b, giving your answer in the form $x + iy$,

(4 marks)

Question 3

Given that $z = x + iy$, find the value of x and the value of y such that

$$z + 3iz^* = -1 + 13i$$

(7 marks)

2.1 Answers

Question 1

$z^* = 2 - 6i$ B1 correct answer

Question 2

$z^* = 1 + 10i$ B1 correct answer

Question 3

$-17 + 7i$ M1: $2i^2 - 3i + 10i - 15$, A1 correct answer

Question 4

a) $a^2 + 2a - 4 + (4a + 4)i$ M1 multiplies out expression (at most 1 error) $a^2 + 4ai + 4i^2 + 2a + 4i$ and correctly using $i^2 = -1$
A1 Correct answer (with i terms collected, not necessarily with I factorised)

b) $a = -1$ (b) and so $4a + 4 = 0 \rightarrow a = -1$ B1

Question 5

$a = \frac{8}{3}$

$8 - 3a = 0$	**M1**	2.2a	Or $2a + 12 + (8 - 3a)i = 2a + 12 - (8 - 3a)i$	Setting imaginary part of **(ii)** to 0.
$a = \frac{8}{3}$	**A1ft**	1.1	or awrt 2.67	

Question 6

$7 - 24i$ M1: expanded to get $16 - 12i - 12i + 9i^2$ and correctly using $i^2 = -1$ A1 correct answer

Question 7

$-46 + 9i$

$2^3 + 3\times2^2\times3i + 3\times2\times(3i)^2 + (3i)^3$	**M1**	1.1	Binomial expansion. Must be 4 terms with 1, 3, 3, 1 soi and correct powers. Condone missing brackets	Or by $(2 + 3i)^2\times(2 + 3i)$ but marks only to awarded once all binomial brackets expanded.
$2^3 + 3\times2^2\times3i - 3\times2\times3^2 - 3^3i$ or better	**A1**	1.1	All correct and $i^2 = -1$ twice.	Must see evidence of i^2 becoming -1 (could be in a table, expanding brackets etc)
$-46 + 9i$	**A1**	1.1		SC if the only working seen is $(-5+12i)(2+3i) = -46+9i$ award B1

Question 8

$z^2 = -5 - 12i$

(a) $(2-3i)(2-3i) = ...$ **Expand and use** $i^2 = -1$, getting completely correct expansion of 3 or 4 terms	M1
Reaches $-5-12i$ after completely correct work (must see $4-9$) (*)	A1cso

Question 9

$a = 3, b = -5$

$z^2 = (2 - i\sqrt{3})(2 - i\sqrt{3})$ $= 4 - 2i\sqrt{3} - 2i\sqrt{3} + 3i^2$	An attempt to multiply out the brackets to give four terms (or four terms implied).	M1
$= 2 - i\sqrt{3} + (4 - 4i\sqrt{3} - 3)$ $= 2 - i\sqrt{3} + (1 - 4i\sqrt{3})$ $= 3 - 5i\sqrt{3}$ (Note: $a = 3, b = -5$.)	M1: An understanding that $i^2 = -1$ and an attempt to add z and put in the form $a + bi\sqrt{3}$ A1: $3 - 5i\sqrt{3}$	M1A1
$z + z^2 = 2 - i\sqrt{3} + (4 - 4i\sqrt{3} + 3) = 9 - 5i\sqrt{3}$ scores M1M0A0 (No evidence of $i^2 = -1$)		

Question 10

$z = 2 - 2i$

(a) $z = \dfrac{4(1-i)}{(1+i)(1-i)}$	M1
$z = 2(1-i)$ or $2-2i$ or exact equivalent.	A1

Question 11

$-5 + 2i$

$\dfrac{-8+9i}{2-i} \times \dfrac{2+i}{2+i}$	M1
$= \dfrac{-16-8i+18i-9}{5} = -5+2i$ i.e. $a = -5$ and $b = 2$ or $-\frac{2}{5}a$	A1 A1ft

Question 12

$a = \frac{3}{5}, b = -\frac{4}{5}$

M1: $\frac{2-i}{2+i} \times \frac{2-i}{2-i}$ A1, A1ft: $\frac{4-4i-1}{4+1} = \frac{3-4i}{5}$

Question 13

$a = \frac{7}{2}, b = \frac{5}{2}$

M1: $\frac{6-i}{1-i} \times \frac{1+i}{1+i}$ A1, A1ft: $\frac{6+5i+1}{1+1} = \frac{7+5i}{2}$

Question 14

$c = 3, d = 2$

$\frac{z+7}{z-1} = \frac{2-\mathrm{i}\sqrt{3}+7}{2-\mathrm{i}\sqrt{3}-1}$	Substitutes $z = 2 - \mathrm{i}\sqrt{3}$ into both numerator and denominator.	M1
$= \frac{(9-\mathrm{i}\sqrt{3})}{(1-\mathrm{i}\sqrt{3})} \times \frac{(1+\mathrm{i}\sqrt{3})}{(1+\mathrm{i}\sqrt{3})}$	Simplifies $\frac{z+7}{z-1}$ **and** multiplies by $\frac{\textbf{their } (1+\mathrm{i}\sqrt{3})}{\textbf{their } (1+\mathrm{i}\sqrt{3})}$	dM1
$= \frac{9+9\mathrm{i}\sqrt{3} - \mathrm{i}\sqrt{3}+3}{1+3}$ $= \frac{12+8\mathrm{i}\sqrt{3}}{4}$	Simplifies realising that a real number is needed in the denominator and applies $\mathrm{i}^2 = -1$ in their numerator expression and denominator expression.	M1
$= 3 + 2\mathrm{i}\sqrt{3}$ (Note: $c = 3$, $d = 2$.)	$3 + 2\mathrm{i}\sqrt{3}$	A1

Question 15

$\frac{2p+12}{13} + \frac{3p-8}{13} i$

$w = \frac{(p-4\mathrm{i})}{(2-3\mathrm{i})} \times \frac{(2+3\mathrm{i})}{(2+3\mathrm{i})}$	Multiplies by $\frac{(2+3\mathrm{i})}{(2+3\mathrm{i})}$	M1
$= \left(\frac{2p+12}{13}\right) + \left(\frac{3p-8}{13}\right)\mathrm{i}$	At least one of either the real or imaginary part of w is correct. Must be expanded but could be unsimplified e.g. expressed as single fraction. Condone $a + \mathrm{i}b$.	A1
	Correct w in its simplest form.	A1

2.2 Answers

Question 1

$w = \frac{3p-10}{6} + \left(\frac{p-12}{6}\right) i$

$\frac{3w+7}{5} = \frac{(p-4\mathrm{i})}{(3-\mathrm{i})} \times \frac{(3+\mathrm{i})}{(3+\mathrm{i})}$	Multiplies by $\frac{(3+\mathrm{i})}{(3+\mathrm{i})}$ or divide by $(9-3\mathrm{i})$ then multiply by $\frac{(9+3\mathrm{i})}{(9+3\mathrm{i})}$	M1
$= \left(\frac{3p+4}{10}\right) + \left(\frac{p-12}{10}\right)\mathrm{i}$	Evidence of $(3-\mathrm{i})(3+\mathrm{i}) = 10$ or 3^2+1^2 or 9^2+3^2	B1
	Rearranges to $w = \ldots$	dM1
So, $w = \left(\frac{3p-10}{6}\right) + \left(\frac{p-12}{6}\right)\mathrm{i}$	At least one of either the real or imaginary part of w is correct in any equivalent form.	A1
	Correct w in the form $a + b\mathrm{i}$. Accept $a + \mathrm{i}b$.	A1

Question 2

$\frac{12a-5b}{13}+\frac{5a+12b}{13}i$

$\frac{z_1z_3}{z_2}=\frac{(2+3i)(a+bi)}{3+2i}$ $=\frac{(2+3i)(a+bi)(3-2i)}{(3+2i)(3-2i)}$	Substitutes for z_1, z_2 and z_3 and multiplies by $\frac{3-2i}{3-2i}$	M1
$(3+2i)(3-2i)=13$	13 seen.	B1
$\frac{z_1z_3}{z_2}=\frac{(12a-5b)+(5a+12b)i}{13}$	M1: Obtains a numerator with 2 real and 2 imaginary parts. A1: As stated or $\frac{(12a-5b)}{13}+\frac{(5a+12b)}{13}i$ ONLY.	dM1A1

Question 3

$x = 5, y = -2$

$z+3iz^*=-1+13i$		
$(x+iy)+3i(x-iy)$	$z^*=x-iy$	B1
	Substituting $z=x+iy$ and their z^* into $z+3iz^*$	M1
$x+iy+3ix+3y=-1+13i$	Correct equation in x and y with $i^2=-1$. Can be implied.	A1
$(x+3y)+i(y+3x)=-1+13i$		
Re part: $x+3y=-1$ Im part: $y+3x=13$	An attempt to equate real **and** imaginary parts.	M1
	Correct equations.	A1
$3x+9y=-3$ $3x+y=13$		
$8y=-16 \Rightarrow y=-2$	Attempt to solve simultaneous equations to find one of x or y. **At least one of the equations must contain both x and y terms.**	M1
$x+3y=-1 \Rightarrow x-6=-1 \Rightarrow x=5$	Both $x=5$ and $y=-2$.	A1