Extension questions – Summer holiday work

Expanding brackets and simplifying expressions

1 Expand and simplify $(x + 3)^2 + (x - 4)^2$

2 Expand and simplify.

   a $\left(x + \dfrac{1}{x}\right)\left(x - \dfrac{2}{x}\right)$ **b** $\left(x + \dfrac{1}{x}\right)^2$

Surds and rationalising the denominator

3 Expand and simplify $\left(\sqrt{x} + \sqrt{y}\right)\left(\sqrt{x} - \sqrt{y}\right)$

4 Rationalise and simplify, if possible.

   a $\dfrac{1}{\sqrt{9} - \sqrt{8}}$ **b** $\dfrac{1}{\sqrt{x} - \sqrt{y}}$

Rules of indices

5 Write as sums of powers of x.

   a $\dfrac{x^5 + 1}{x^2}$ **b** $x^2\left(x + \dfrac{1}{x}\right)$ **c** $x^{-4}\left(x^2 + \dfrac{1}{x^3}\right)$

Factorising expressions

6 Simplify $\sqrt{x^2 + 10x + 25}$

7 Simplify $\dfrac{(x + 2)^2 + 3(x + 2)^2}{x^2 - 4}$

Completing the square

8 Write the following quadratic expressions in the form $p(x + q)^2 + r$
   a $2x^2 - 8x - 16$ **b** $4x^2 - 8x - 16$
   c $3x^2 + 12x - 9$ **d** $2x^2 + 6x - 8$

9 Complete the square.
   a $2x^2 + 3x + 6$ **b** $3x^2 - 2x$
   c $5x^2 + 3x$ **d** $3x^2 + 5x + 3$

10 Write $(25x^2 + 30x + 12)$ in the form $(ax + b)^2 + c$.

Solving linear simultaneous equations using the elimination method

11 Solve the simultaneous equations $3x + 5y - 20 = 0$ and $2(x + y) = \dfrac{3(y - x)}{4}$.

Solving linear and quadratic simultaneous equations

12 $x - y = 1$

$x^2 + y^2 = 3$

13 $y - x = 2$

$x^2 + xy = 3$

Linear inequalities

14 Find the set of values of x for which $2x + 1 > 11$ and $4x - 2 > 16 - 2x$.

Straight line graphs

15 The equation of a line is $2y + 3x - 6 = 0$.
Write as much information as possible about this line.

Parallel and perpendicular lines

16 Work out whether these pairs of lines are parallel, perpendicular or neither.

a $y = 2x + 3$
$y = 2x - 7$

b $y = 3x$
$2x + y - 3 = 0$

c $y = 4x - 3$
$4y + x = 2$

d $3x - y + 5 = 0$
$x + 3y = 1$

e $2x + 5y - 1 = 0$
$y = 2x + 7$

f $2x - y = 6$
$6x - 3y + 3 = 0$

17 The straight line $\mathbf{L_1}$ passes through the points A and B with coordinates $(-4, 4)$ and $(2, 1)$, respectively.

a Find the equation of $\mathbf{L_1}$ in the form $ax + by + c = 0$

The line $\mathbf{L_2}$ is parallel to the line $\mathbf{L_1}$ and passes through the point C with coordinates $(-8, 3)$.

b Find the equation of $\mathbf{L_2}$ in the form $ax + by + c = 0$

The line $\mathbf{L_3}$ is perpendicular to the line $\mathbf{L_1}$ and passes through the origin.
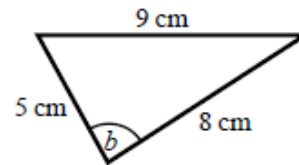
c Find an equation of $\mathbf{L_3}$

Trigonometry in right-angled triangles

18   Find the size of each lettered angle or side.
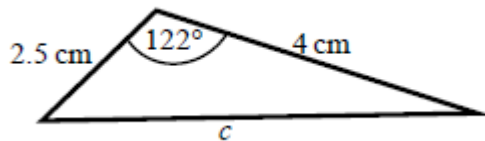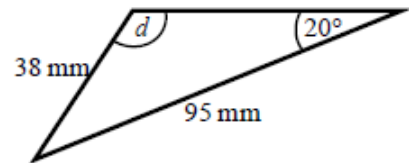Give your answers correct to 3 significant figures.

a

7.5 cm
a
52°
70°

b

9 cm
5 cm
8 cm
b

c

2.5 cm
122°
4 cm
c

d

d
20°
38 mm
95 mm
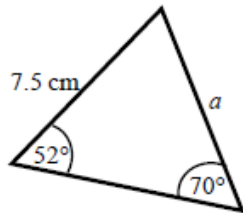
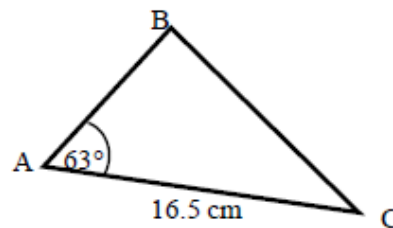**19**   The area of triangle ABC is 86.7 cm².
Work out the length of BC.
Give your answer correct to 3 significant figures.

B
A 63°
16.5 cm
C

Rearranging equations

20   Make *x* the subject of the following equations.

a $\dfrac{p}{q}(sx+t) = x-1$ **b** $\dfrac{p}{q}(ax+2y) = \dfrac{3p}{q^2}(x-y)$

Answers - Extension questions – Summer holiday work

1 $2x^2 - 2x + 25$

2   **a** $x^2 - 1 - \dfrac{2}{x^2}$

      b $x^2 + 2 + \dfrac{1}{x^2}$

3 $x - y$

4   **a** $3 + 2\sqrt{2}$

      b $\dfrac{\sqrt{x} + \sqrt{y}}{x - y}$

5   **a** $x^3 + x^{-2}$

      b $x^3 + x$

      c $x^{-2} + x^{-7}$

6 $(x + 5)$

7 $\dfrac{4(x + 2)}{x - 2}$

8   **a** $2(x - 2)^2 - 24$

      b $4(x - 1)^2 - 20$

      c $3(x + 2)^2 - 21$

      d $2\left(x + \dfrac{3}{2}\right)^2 - \dfrac{25}{2}$

9   **a** $2\left(x + \dfrac{3}{4}\right)^2 + \dfrac{39}{8}$

      b $3\left(x - \dfrac{1}{3}\right)^2 - \dfrac{1}{3}$

      c $5\left(x + \dfrac{3}{10}\right)^2 - \dfrac{9}{20}$

      d $3\left(x + \dfrac{5}{6}\right)^2 + \dfrac{11}{12}$

10 $(5x + 3)^2 + 3$

11 $x = -2\dfrac{1}{2},\ y = 5\dfrac{1}{2}$

12 $x = \dfrac{1 + \sqrt{5}}{2},\ y = \dfrac{-1 + \sqrt{5}}{2}$

$$x = \frac{1-\sqrt{5}}{2}, \ y = \frac{-1-\sqrt{5}}{2}$$

13 $\quad x = \frac{-1+\sqrt{7}}{2}, \ y = \frac{3+\sqrt{7}}{2}$

$\qquad x = \frac{-1-\sqrt{7}}{2}, \ y = \frac{3-\sqrt{7}}{2}$

14 $\quad x > 5$ (which also satisfies $x > 3$)

15 $\quad y = -\frac{3}{2}x + 3$, the gradient is $-\frac{3}{2}$ and the y-intercept is 3.

The line intercepts the axes at (0, 3) and (2, 0).

Students may sketch the line or give coordinates that lie on the line such as $\left(1, \frac{3}{2}\right)$ or $\left(4, -3\right)$.

16 **a** Parallel **b** Neither **c** Perpendicular

$\qquad$ **d** Perpendicular **e** Neither **f** Parallel

17 **a** $x + 2y - 4 = 0$ **b** $x + 2y + 2 = 0$ **c** $y = 2x$

18 **a** 6.29 cm **b** 84.3° **c** 5.73 cm **d** 58.8°

19 15.3 cm

20 **a** $x = \frac{q + pt}{q - ps}$ **b** $x = \frac{3py + 2pqy}{3p - apq} = \frac{y(3 + 2q)}{3 - aq}$