A Level Further Maths

Pearson Edexcel Level 3 Advanced GCE in Further Mathematics (9FM0)

Core Pure Mathematics (Paper 1 & 2): 50% of the qualification
Further Mechanics 1 (Paper 3): 25% of the qualification
Decision Mathematics 1 (Paper 4): 25% of the qualification

Books you will be provided with:

Pearson: Core Pure Mathematics Book 1; Core Pure Mathematics Book 2; Further Mechanics 1; Decision Mathematics 1.

Resources you may choose to buy/look at (although these are entirely optional):
None, really. However, there are many excellent Maths themed books available for your reading pleasure! (see summer work reading list)

Work to be completed by 18th July:

1. Neat notes on Complex Numbers (use textbook pages to help)
   To include:
   - An explanation of each concept
   - An example (or two) for each concept
   - Any key facts/techniques to remember
2. First three questions from the Taster Day booklet of questions
   - Completed
   - Checked using the answers (these only include a final answer and not a method)
3. Decide on a Maths themed book/article you are going to read (see reading list for suggestions)

Work to be completed by September:

1. Complete the remaining questions from the Taster Day booklet of questions
   - Completed
   - Checked using the answers (these only include a final answer and not a method)
2. A paragraph or two of something interesting you have read

What to bring with you to the first lesson in September:

All of the above work.

Chaos

- **Does God Play Dice** by *Ian Stewart*

An introduction to chaotic dynamics. Doesn't contain that many actual sums, but lots of pretty pictures, and a good overview of the role of chaos. Quite a popularist style.

- **Chaos** by *James Gleick*

Quite Physicsy, but a good read, yet again quite biographical, some have said that it gets hard work to read quite soon after opening!

Cryptography

- **The Codebook** by *Simon Singh*

Interesting exploration into the different types of codes and CYPHERS used throughout history. Is a very good GENERAL MATHS BOOK, covering elements of basic number theory, physics (potential of photon money!), statistics (frequency Analysis) and computing.

- **The Mathematics of Ciphers** by *S.C. Coutinho*
- **In Code** by *Sara Flannery*

History of Mathematics

- **A History of Mathematics** by *Carl B. Boyer*
- **Infinity: The Quest to Think the Unthinkable** by *Brian Clegg*

Am currently reading this. This is definitely one of the better books on the subject. A chronological biography of the concept of infinity, from Greeks to present day.

- **E, the Story of a Number** by *Eli Maor*

Biographies

- **The Man Who Loved Only Numbers** by *Paul Hoffman*

An excellent account of one of the 20th Century's most prolific mathematicians.

- **The Man who knew Infinity** by *Robert Kanigel*

Book about Ramanujan, yet again more biographical, but still worth a look.

- **Abel's Proof: An Essay on the Sources and Meaning of Mathematical Unsolvability** by *Peter Pesic*

Mathematical Physics

- **A Brief History of Time** by *Stephen Hawking*
- **The Elegant Universe** by *Brian Greene*

A book about string theory, but most of the book is about relativity and quantum mechanics etc.

- **The Fabric of the Cosmos** by *Brian Greene*

Sequel to the above. Focuses more on new research. Both books are very interesting.

Mathematical Philosophy

- **Introduction to Mathematical Philosophy** by *Bertrand Russell*

Mathematical Problems

- **Fermat's Last Theorem** by *Simon Singh*

  Very popular book!
- **The Millenium Problems** by *Keith Devlin*
- **Journey Through Genius: The Great Theorems of Mathematics** by *William Dunham*
- **The Equation That Couldn't Be Solved** by *Mario Livio*
- **Kepler's Conjecture** by *George Szpiro*
- **Poincaré's Prize** by *George Szpiro*
- **The Music of the Primes** by *Marcus du Sautoy*

About the Riemann hypothesis and other various topics in number theory.

Four Colors Suffice by *Robin Wilson*

1. $f(x) = 2x^3 - 8x^2 + 7x - 3$

Given that $x = 3$ is a solution of the equation $f(x) = 0$, solve $f(x) = 0$ completely.

(5)

9. Given that $z_1 = 3 + 2i$ and $z_2 = \dfrac{12 - 5i}{z_1}$,

(a) find z_2 in the form $a + ib$, where a and b are real.

(2)

(b) Show on an Argand diagram the point P representing z_1 and the point Q representing z_2.

(2)

1. The complex numbers z_1 and z_2 are given by

$$z_1 = 2 - i \quad \text{and} \quad z_2 = -8 + 9i$$

(a) Show z_1 and z_2 on a single Argand diagram.

(1)

Find, showing your working,

(b) the value of $|z_1|$,

(2)

(c) the value of $\arg z_1$, giving your answer in radians to 2 decimal places,

(2)

(d) $\dfrac{z_2}{z_1}$ in the form $a + bi$, where a and b are real.

(3)

3. $f(x) = (x^2 + 4)(x^2 + 8x + 25)$

(a) Find the four roots of $f(x) = 0$.

(5)

(b) Find the sum of these four roots.

(2)

1. The complex numbers z_1 and z_2 are given by

$$z_1 = 2 + 8i \quad \text{and} \quad z_2 = 1 - i$$

Find, showing your working,

(a) $\frac{z_1}{z_2}$ in the form $a + bi$, where a and b are real,

(3)

(b) the value of $\left|\frac{z_1}{z_2}\right|$,

(2)

(c) the value of $\arg \frac{z_1}{z_2}$, giving your answer in radians to 2 decimal places.

(2)

6. Given that 2 and $5 + 2i$ are roots of the equation

$$x^3 - 12x^2 + cx + d = 0, \qquad c, d \in \mathbb{R},$$

(a) write down the other complex root of the equation.

(1)

(b) Find the value of c and the value of d.

(5)

(c) Show the three roots of this equation on a single Argand diagram.

(2)

1.

$$z = 5 - 3i, \qquad w = 2 + 2i$$

Express in the form $a + bi$, where a and b are real constants,

(a) z^2,

(2)

(b) $\frac{z}{w}$.

(3)

4. Given that $2 - 4i$ is a root of the equation

$$z^2 + pz + q = 0,$$

where p and q are real constants,

(a) write down the other root of the equation,

(1)

(b) find the value of p and the value of q.

(3)

1. Given that $z_1 = 1 - i$,

(a) find $\arg(z_1)$.

(2)

Given also that $z_2 = 3 + 4i$, find, in the form $a + ib$, $a, b \in \mathbb{R}$,

(b) $z_1 z_2$,

(2)

(c) $\frac{z_2}{z_1}$.

(3)

In part (b) and part (c) you must show all your working clearly.

2.

$$z = \frac{50}{3 + 4i}$$

Find, in the form $a + ib$ where $a, b \in \mathbb{R}$,

(a) z,

(2)

(b) z^2.

(2)

Find

(c) $|z|$,

(2)

(d) $\arg z^2$, giving your answer in degrees to 1 decimal place.

(2)

1. $x = \dfrac{1 \pm \mathrm{i}}{2}$

9. (*a*) $z_2 = 2 - 3\,\mathrm{i}$

1. (*b*) $\sqrt{5}$ (*c*) -0.46 or 5.82 (*d*) $-5 + 2\mathrm{i}$

3. (*a*) $x = k\mathrm{i}$; $x = 2\mathrm{i}$; $x = -4 + 3\mathrm{i}$; $x = -4 - 3\mathrm{i}$

(*b*) -8

1. (*a*) $-3 + 5\mathrm{i}$ (*b*) $\sqrt{34}$ (*c*) 2.11

6. (*a*) $5 - 2\mathrm{i}$ (*b*) $c = 49,\ d = -58$

1. (*a*) $16 - 30\mathrm{i}$ (*b*) $\dfrac{1}{2} - 2\mathrm{i}$

4. (*a*) $2 + 4\mathrm{i}$ (*b*) $p = -4,\ q = 20$

1. (*a*) $-\dfrac{\pi}{4}$ (*b*) $7 + \mathrm{i}$ (*c*) $\dfrac{1}{2} + \dfrac{7}{2}\mathrm{i}$

2. (*a*) $z = 6 - 8\mathrm{i}$ (*b*) $z^2 = -28 - 96\mathrm{i}$

(*c*) $|z| = 10$ (*d*) $a = -106.3°$ or $253.7°$